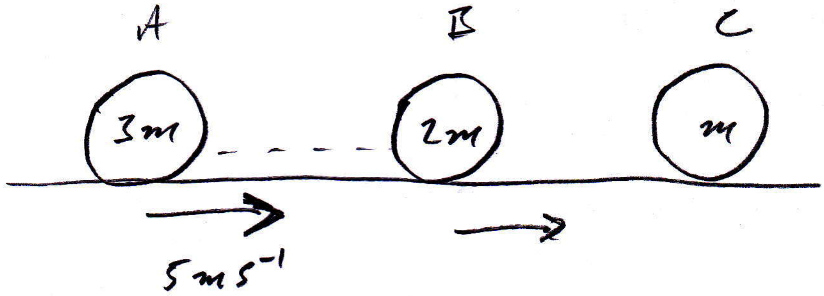
2012 Q5.

(a) (Long)



A → B:

PCM $\quad 3m(5) + 2m(0) = 3mV_1 + 2mV_2$ ①

NEL $\quad \dfrac{V_1 - V_2}{5 - 0} = -e$ ②

① $\quad 15m = 3mV_1 + 2mV_2$

$\Rightarrow 15 = 3V_1 + 2V_2 \Rightarrow V_1 = \dfrac{15 - 2V_2}{3}, \quad V_2 = \dfrac{15 - 3V_1}{2}$

② $\quad V_1 - V_2 = -5e$

$\Rightarrow \dfrac{15 - 2V_2}{3} - V_2 = -5e$

$\Rightarrow 15 - 2V_2 - 3V_2 = -15e$

$\Rightarrow -5V_2 = -15e - 15$

$\Rightarrow V_2 = 3e + 3$

Similarly: $V_1 = 3 - 2e$

B → C:

PCM $\quad 2mV_2 + m(0) = 2mV_3 + mV_4$

$\Rightarrow 2V_2 = 2V_3 + V_4$ ③

NEL $\quad \dfrac{V_3 - V_4}{V_2 - 0} = -e \Rightarrow V_3 - V_4 = -V_2 e$ ④

$\Rightarrow V_4 = V_2 e + V_3$ and $V_3 = V_4 - V_2 e$

$$2V_2 = 2V_3 + V_4 \quad (3)$$

$$\Rightarrow 2V_3 = 2V_2 - V_4$$

$$\Rightarrow 2V_3 = 2V_2 - V_2 e - V_3$$

$$\Rightarrow 3V_3 = 2V_2 - V_2 e$$

$$\Rightarrow 3V_3 = V_2(2-e)$$

$$\Rightarrow 3V_3 = (3e+3)(2-e)$$

$$\Rightarrow V_3 = (e+1)(2-e) \quad *$$

$$2V_2 = 2V_3 + V_4 \quad (3)$$

$$\Rightarrow V_4 = 2V_2 - 2V_3$$

$$\Rightarrow V_4 = 2(3e+3) - 2(V_4 - V_2 e)$$

$$\Rightarrow V_4 = 6e + 6 - 2V_4 + 2V_2 e$$

$$\Rightarrow 3V_4 = 6e + 6 + 2(3e+3)e$$

$$\Rightarrow 3V_4 = 2(3e+3) + 2e(3e+3)$$

$$\Rightarrow 3V_4 = (2+2e)(3e+3)$$

$$\Rightarrow 3V_4 = 6(1+e)(e+1)$$

$$\Rightarrow V_4 = 2(1+e)(e+1) \quad *$$

For no further impacts $V_1 < V_3 < V_4$

$V_1 < V_3 \Rightarrow (3-2e) < (1+e)(2-e)$

$\Rightarrow \quad 3 - 2e < 2 - e + 2e - e^2$

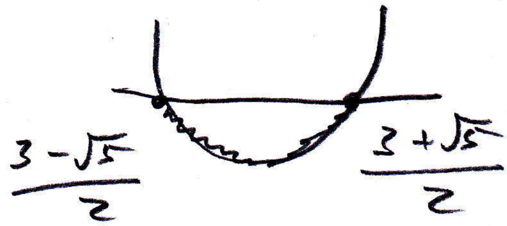
$\Rightarrow \quad e^2 - 3e + 1 < 0$

Let $e^2 - 3e + 1 = 0$ $\qquad a = 1$

$\qquad b = -3$

$$e = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \qquad c = 1$$

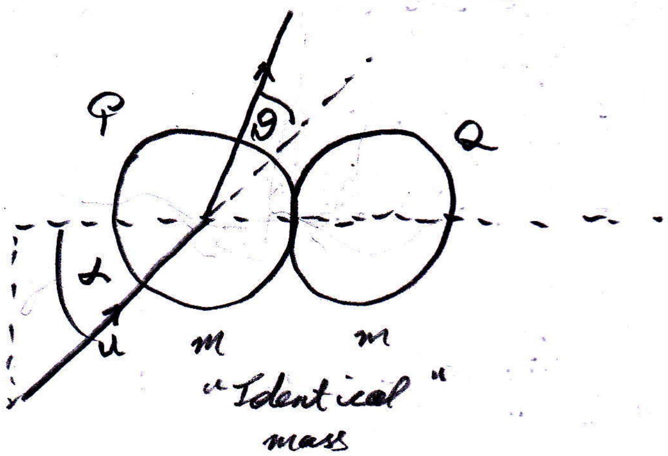
$$= \frac{3 \pm \sqrt{5}}{2}$$



$$\Rightarrow \quad \frac{3-\sqrt{5}}{2} < e < \frac{3+\sqrt{5}}{2}$$

$$\Rightarrow \quad e > \frac{3-\sqrt{5}}{2}$$

(b)



$e = \frac{1}{3}$

PCM: $\quad m(u\cos\alpha) + m(0) = mV_1 + mV_2$

$\Rightarrow \quad u\cos\alpha = V_1 + V_2$

NEL: $\quad V_1 - V_2 = -\frac{1}{3}(u\cos\alpha - 0)$

Adding: $\Rightarrow \quad 2V_1 = -\frac{1}{3}u\cos\alpha + u\cos\alpha$

$$\Rightarrow \quad 2V_1 = \frac{2}{3} u \cos \alpha$$

$$\Rightarrow \quad V_1 = \frac{1}{3} u \cos \alpha$$

$$\tan(\alpha + \theta) = \frac{u \sin \alpha}{V_1}$$

$$\frac{\tan \alpha + \tan \theta}{1 - \tan \alpha \tan \theta} = \frac{u \sin \alpha}{\frac{1}{3} u \cos \alpha}$$

$$= \frac{3 u \sin \alpha}{u \cos \alpha}$$

$$\frac{\tan \alpha + \tan \theta}{1 - \tan \alpha \tan \theta} = \frac{3 \tan \alpha}{1}$$

Note: $u \sin \alpha$ is the velocity in the j direction which remains unchanged after impact.

V_1 = velocity P in the $\vec{i}$ direction after the collision.

(See p118)

$$\Rightarrow 3 \tan \alpha - 3 \tan^2 \alpha \tan \theta = \tan \alpha + \tan \theta$$

$$\Rightarrow 3 \tan \alpha - \tan \alpha = 3 \tan^2 \alpha \tan \theta + \tan \theta$$

$$\Rightarrow \quad 2 \tan \alpha = \tan \theta \left(3 \tan^2 \alpha + 1\right)$$

$$\Rightarrow \quad \frac{2 \tan \alpha}{3 \tan^2 \alpha + 1} = \tan \theta$$