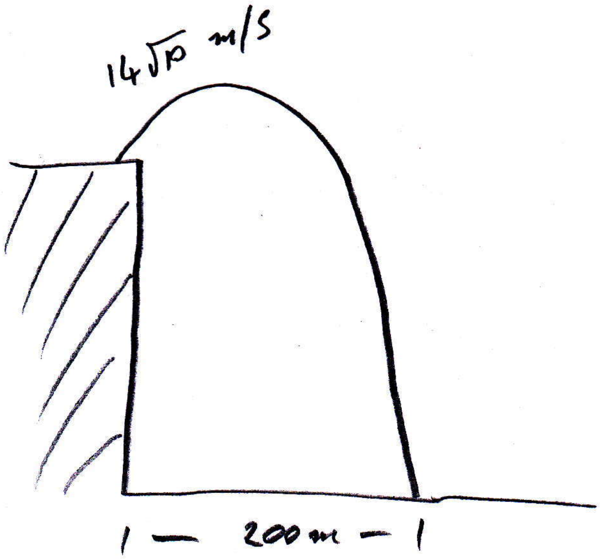
2009 Q3.

(a)



(i)

$$S_x = U_x t$$

$$\Rightarrow 200 = 14\sqrt{10}\, \text{Cos}\,\alpha\, t$$

$$\Rightarrow t = \frac{200}{14\sqrt{10}\, \text{Cos}\,\alpha}$$

<u>Rem:</u> $\text{Sec}^2 = 1 + \text{Tan}^2$

(ii) $$S_y = U_y t - \tfrac{1}{2} g t^2$$

$$\Rightarrow -200 = 14\sqrt{10}\, \text{Sin}\,\alpha\, t - \tfrac{1}{2} g t^2$$

$$\Rightarrow -200 = 14\sqrt{10}\, \text{Sin}\,\alpha \left(\frac{200}{14\sqrt{10}\, \text{Cos}\,\alpha}\right) - \frac{1}{2} g \left(\frac{200}{14\sqrt{10}\, \text{Cos}\,\alpha}\right)^2$$

$$\Rightarrow -200 = 200\, \text{Tan}\,\alpha - \frac{1}{2} g \left(\frac{40000}{1960\, \text{Cos}^2\alpha}\right)$$

$$\Rightarrow -200 = 200\, \text{Tan}\,\alpha - 100 \frac{1}{\text{Cos}^2\alpha}$$

$$\Rightarrow -200 = 200\, \text{Tan}\,\alpha - 100(1 + \text{Tan}^2\alpha)$$

$$\Rightarrow -2 = 2\, \text{Tan}\,\alpha - 1 - \text{Tan}^2\alpha$$

$$\Rightarrow 0 = \text{Tan}^2\alpha - 2\, \text{Tan}\,\alpha - 1$$

$\Rightarrow$

Let $y = \tan\alpha \quad \Rightarrow \quad y^2 - 2y - 1 = 0$

$a = 1$
$b = -2$
$c = -1$

$$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 \pm \sqrt{2}$$

$\Rightarrow \tan\alpha = 1 + \sqrt{2}$ or $\tan\alpha = 1 - \sqrt{2}$

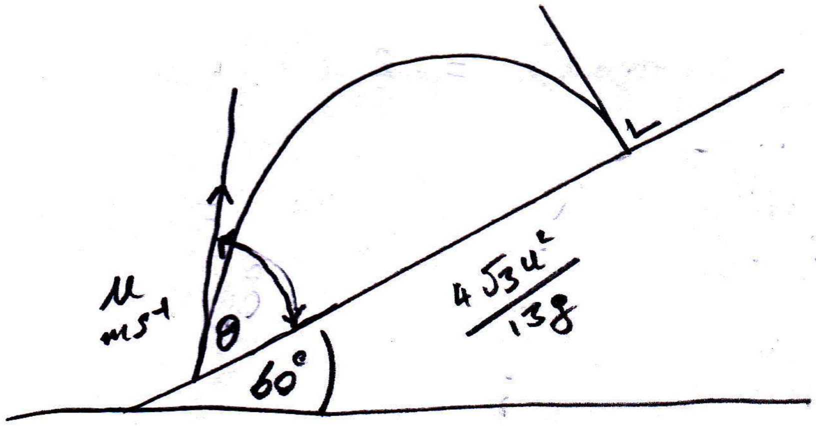
Note: Tan is "slope" of line
If both directions of projection are perpendicular then the product of their slopes is -1

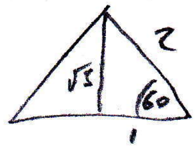
$$(1+\sqrt{2})(1-\sqrt{2}) = 1 - \sqrt{2} + \sqrt{2} - 2$$

$$= -1$$

$\Rightarrow$ directions are perpendicular.

(b)





$$S_y = u\sin\theta\, t - \tfrac{1}{2} g \cos 60^\circ\, t^2$$

$$\Rightarrow \quad 0 = u\sin\theta\, t - \tfrac{1}{2} g \tfrac{1}{2} t^2$$

$$\Rightarrow \quad 0 = t\left(u\sin\theta - \tfrac{1}{4} g t\right)$$

$$\Rightarrow \quad u\sin\theta - \tfrac{1}{4} g t = 0$$

$$\Rightarrow \quad t = \frac{4u\sin\theta}{g} \qquad *$$

$$V_x = u\cos\theta - g\sin 60\, t$$

$$\Rightarrow \quad 0 = u\cos\theta - g\frac{\sqrt{3}}{2} t$$

$$\Rightarrow \quad t = \frac{2u\cos\theta}{g\sqrt{3}} \qquad *$$

* Equating both t's:

$$\frac{2u\cos\theta}{g\sqrt{3}} = \frac{4u\sin\theta}{g}$$

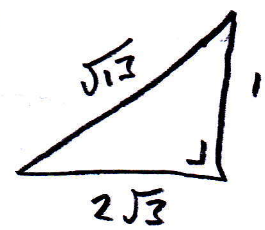
$$\Rightarrow \quad 4\sqrt{3}\, u \sin\theta = 2u \cos\theta$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{2}{4\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{2\sqrt{3}}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{13}}$$

$$\text{and } \cos\theta = \frac{2\sqrt{3}}{\sqrt{13}}$$



$$\sqrt{(2\sqrt{3})^2 + 1^2}$$

$$= \sqrt{13}$$

$$\text{Range} = S_x = u\cos\theta\, t - \tfrac{1}{2} g \sin 60^\circ\, t^2$$

$$= u\, \frac{2\sqrt{3}}{\sqrt{13}} \left(\frac{4u\sin\theta}{g} \right) - \frac{1}{2} g\, \frac{\sqrt{3}}{2} \left(\frac{4u\sin\theta}{g} \right)^2$$

$$= \frac{8\sqrt{3}\, u^2}{g\sqrt{13}} \cdot \frac{1}{\sqrt{13}} - \frac{\sqrt{3}}{4} g \left(\frac{16u^2 \left(\frac{1}{\sqrt{13}}\right)^2}{g^2} \right)$$

$$= \frac{8\sqrt{3}\, u^2}{13g} - \frac{4\sqrt{3}\, u^2}{13g}$$

$$= \frac{4\sqrt{3}\, u^2}{13g}$$