2012 Q8.

(a) See 2013 Q8(a)

(b)(i) $mgh_1 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega_1^2 = mgh_2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

(Mass) (Mass) (Disc) Mass Mass Disc.

$\Rightarrow mgh_1 + \frac{1}{2}m(0)^2 + \frac{1}{2}I(0)^2 = mg(0) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Note: $v = \omega r \Rightarrow v^2 = \omega^2 r^2$

$\Rightarrow \frac{v^2}{r^2} = \omega^2$

$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$ ①

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = 0^2 + 2ah$

$\Rightarrow v^2 = 2ah$ ②

Substituting ② into ①

$\Rightarrow mgh = \frac{1}{2}m(2ah) + \frac{1}{2}\frac{I(2ah)}{r^2}$

$\Rightarrow mg = a\left(m + \frac{I}{r^2}\right)$

$\Rightarrow a = \frac{mg}{m + \frac{I}{r^2}} \Rightarrow a = \frac{mgr^2}{I + mr^2}$

$$F = ma$$

$$\Rightarrow mg - T = ma$$

$$\Rightarrow T = mg - ma$$

$$\Rightarrow T = mg - m\left(\frac{mgr^2}{I + mr^2}\right)$$

$$\Rightarrow T = mg\left(1 - \frac{mr^2}{I + mr^2}\right)$$

$$\Rightarrow T = mg\left(\frac{I + mr^2 - mr^2}{I + mr^2}\right)$$

$$\Rightarrow T = \frac{mgI}{I + mr^2}$$

(ii)

$$\frac{mgr^2}{I + mr^2} = \frac{g}{5}$$

$$\Rightarrow 5mgr^2 = gI + gmr^2$$

$$\Rightarrow 5mr^2 = I + mr^2$$

$$\Rightarrow 5mr^2 - mr^2 = I$$

$$\Rightarrow 4mr^2 = I$$

$$\Rightarrow 4mr^2 = \frac{1}{2}Mr^2$$

$$\Rightarrow 8m = M$$

$$\boxed{I = \frac{1}{2}Mr^2}$$