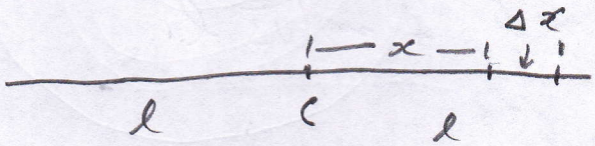
2009 Q8.

(a) $\rho = \frac{\text{mass}}{\text{length}} \Rightarrow \text{Mass} = \rho \text{ length}$

$m = \rho 2l$

Break the rod up into infinitesimally small pieces of length Δx where x is the distance between the piece and C



$\Delta m = \rho \Delta x$

$I = \sum \Delta m r^2$

$= \sum \rho \Delta x \, x^2$

$= \sum \rho x^2 \Delta x$

$= \int_{-l}^{l} \rho x^2 \, dx$

$= \left[\rho \frac{x^3}{3} \right]_{-l}^{l}$

$= \rho \frac{l^3}{3} - \frac{\rho (-l)^3}{3}$

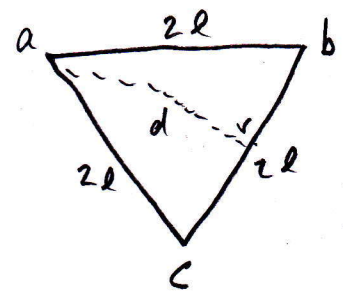
$= \frac{2\rho l^3}{3}$

$= 2\rho l \frac{l^2}{3}$

$= \frac{ml^2}{3}$, since $m = 2\rho l$

$= \frac{1}{3} m l^2$

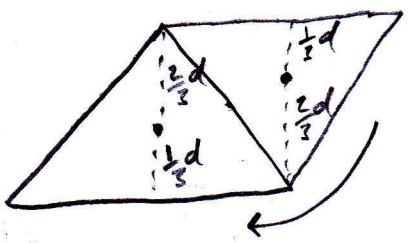
(b) (i)



$$d^2 = (2\ell)^2 - \ell^2$$
$$= 4\ell^2 - \ell^2$$
$$\Rightarrow d = \sqrt{3}\,\ell$$

$$I = \frac{4}{3}m\ell^2 + \frac{4}{3}m\ell^2 + \left(\frac{1}{3}m\ell^2 + md^2\right)$$
$$= \frac{8}{3}m\ell^2 + \frac{1}{3}m\ell^2 + m(\sqrt{3}\,\ell)^2$$
$$= 3m\ell^2 + 3m\ell^2$$
$$= 6m\ell^2$$

(ii) Gain in KE = Loss in PE



$h = \frac{1}{3}d$

$$\frac{1}{2}I\omega^2 = Mgh$$
$$= 3mgh.$$

$$\Rightarrow \frac{1}{2}6m\ell^2\omega^2 = 3mg\left(\frac{1}{3}d\right)$$

$$\Rightarrow \ell^2\omega^2 = \frac{gd}{3}$$

$$\Rightarrow \omega^2 = \frac{gd}{3\ell^2}$$

$$\Rightarrow \omega^2 = \frac{g\sqrt{3}\,\ell}{3\ell^2} \quad [d = \sqrt{3}\,\ell]$$

$$\Rightarrow \omega^2 = \frac{g\sqrt{3}}{3\ell} \quad \text{or} \quad \frac{g}{\sqrt{3}\,\ell} \Rightarrow \omega = \sqrt{\frac{g}{\sqrt{3}\,\ell}}$$

$$\Rightarrow \omega = \sqrt{\frac{g\sqrt{3}}{3\ell}}$$