Junior Certificate Applied Arithmetic

3.3 Applied arithmetic	Solving problems involving, e.g., mobile phone tariffs, currency transactions, shopping, VAT and meter readings. Making value for money calculations and judgments. Using ratio and proportionality.	– solve problems that involve finding profit or loss, % profit or loss (on the cost price), discount, % discount, selling price, compound interest for not more than 3 years, income tax (standard rate only), net pay (including other deductions of specified amounts) **– solve problems that involve cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) compound interest, income tax and net pay (including other deductions)**

Leaving Certificate Arithmetic

3.1 Number Systems

LC Syllabus Page 26

- **Z** of integers
- **Q** of rational numbers
- **R** of real numbers

and represent these numbers on a number line

– appreciate that processes can generate sequences of numbers or objects
– investigate patterns among these sequences
– use patterns to continue the sequence
– generate rules/formulae from those patterns
– develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place value understanding
– consolidate their understanding of factors, multiples, prime numbers in **N**
– express numbers in terms of their prime factors
– appreciate the order of operations, including brackets
– express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{N}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form

– interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate
– generalise and explain patterns and relationships in algebraic form
– recognise whether a sequence is arithmetic, geometric or neither
– find the sum to n terms of an arithmetic series
– express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form

– investigate geometric sequences and series
– prove by induction
- simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series
- simple inequalities such as
  $n! > 2^n$
  $2^n > n^2 \ (n \geq 4)$
  $(1+x)^n \geq 1+nx \ (x > -1)$
- factorisation results such as 3 is a factor of $4^n - 1$

– apply the rules for sums, products, quotients of limits
– find by inspection the limits of sequences such as

$$\lim_{n \to \infty} \frac{n}{n+1}; \quad \lim_{n \to \infty} r^n \ |r|<1$$

– solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment
– derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums

Leaving Certificate Arithmetic

3.3 Arithmetic	– check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – make and justify estimates and approximations of calculations; calculate percentage error and tolerance – calculate average rates of change (with respect to time) – solve problems involving • finding depreciation (reducing balance method) • costing: materials, labour and wastage • metric system; change of units; everyday imperial units (conversion factors provided for imperial units) – estimate of the world around them, e.g. how many books in a library	– accumulate error (by addition or subtraction only) – solve problems that involve calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price), compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions)	– use *present value* when solving problems involving loan repayments and investments

LC Syllabus Page 26

Resources

Students' CD

Teaching & Learning Plans

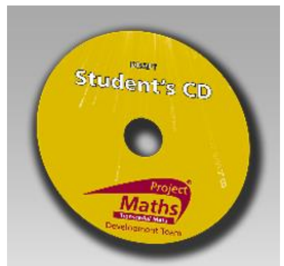
Teacher Handbooks

Algebra

Leaving Certificate Strand 3
Number

Series and Sequences

- Arithmetic Sequence and Series
- Student Activity Arithmetic Sequence and Series
- Arithmetic Sequence Quiz
- Arithmetic Series Quiz
- Geometric Sequence and Series
- Student Activity Sequence and Series
- Geometric Sequence Quiz
- Geometric Series Quiz
- Sum to infinity of a GP



Arithmetic

- ESB Bill
- Student Activity ESB Bill
- VAT
- Student Activity on VAT
- Exchange Rate
- Student Activity on Exchange Rate
- Amortisation of a loan

http://www.projectmaths.ie

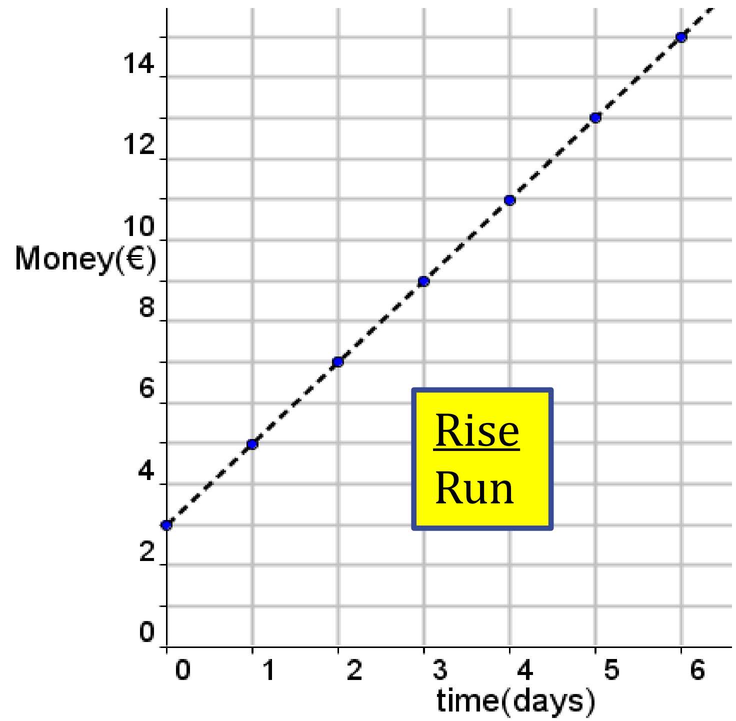
Workshop 4: Patterns and Linear Relationships – 1st year

Multi-representational approach

- Variables
- Constants
- Inputs
- Start amount
- Outputs
- Rate of change

Money Box Problem

Time/days	Money/€	Change
0	3	
1	5	+2
2	7	+2
3	9	+2
4	11	+2
5	13	+2
6	15	+2



$$total\ amount = start\ amount + (rate\ of\ change) \times (number\ of\ days)$$

$$\text{total amount} = 3 + (2 \times \text{number of days})$$

$$A = 3 + 2 \times d$$

$$A = 3 + 2d$$

Local Property Tax- Budget 2013

Local property tax (LPT)

"Interest will be charged on deferred amounts at **4 per cent simple interest per annum**, which is half the rate charged in default cases"

Property: €200,000
Tax: €175,000(0.0018)=€315

$$y = 315 + 12.6t$$

http://www.mccarthyaccountants.com/financebill2012/

Workshop 4: Exponential Relationships

Multi-representational approach

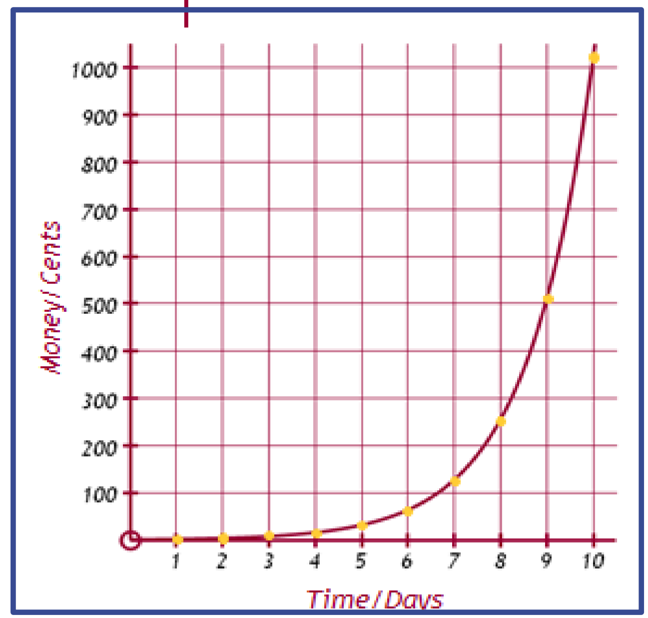
Day	Money in cent
1	2
2	2 x 2
3	2 x 2 x 2
4	

Formula: *y = 2* $y = 2^x$

Words: Doubling

- **Variables and constants**
- **Rate of change**

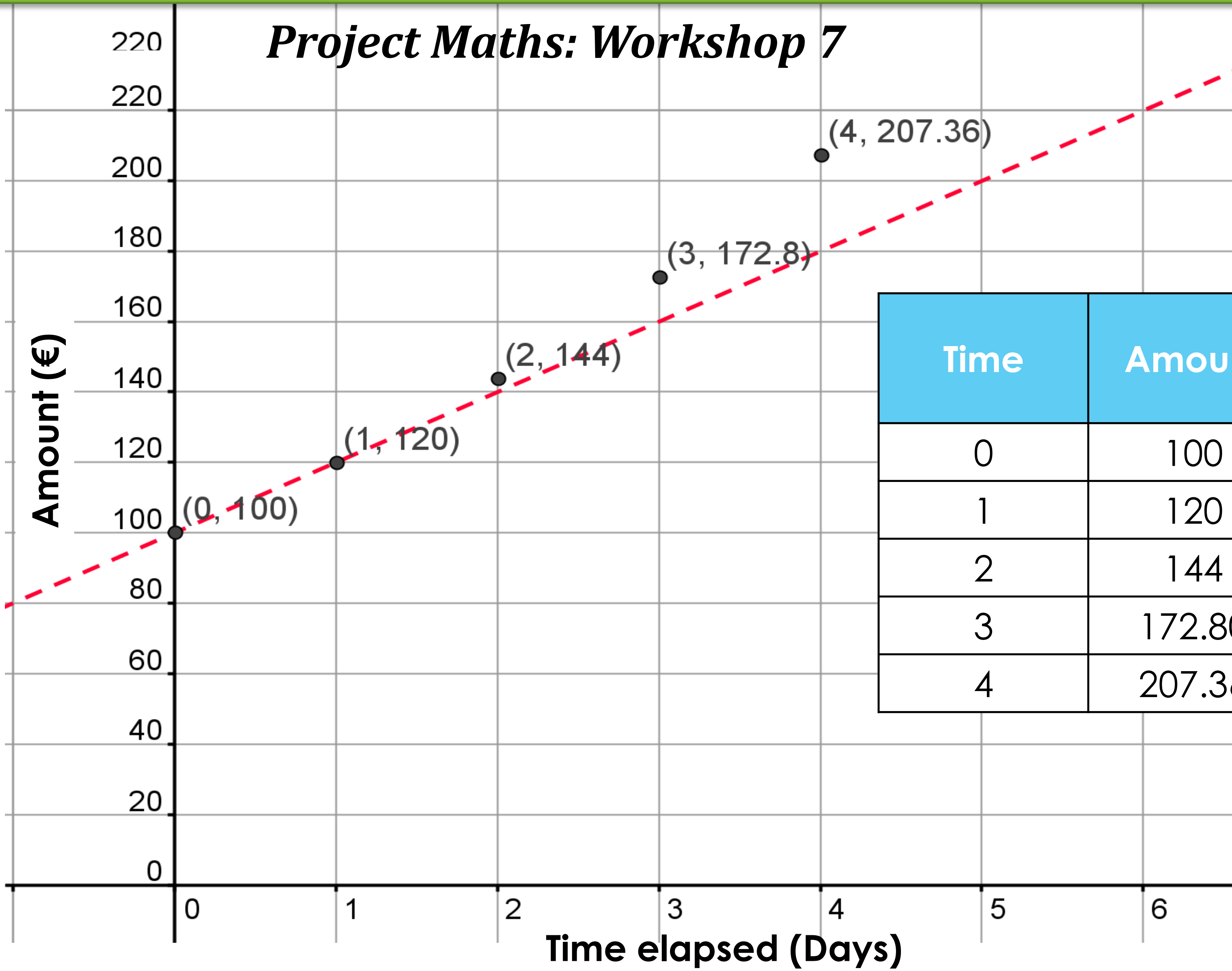
Days	Money in cent	Change	Change of change
0	1	+1	+1
1	2	+2	+2
2	4	+4	+4
3	8	+8	+8
4	16	+16	+16
5	32	+32	+32
6	64	+64	+64
7	128	+128	+128
8	256	+256	
9	512		



There is a constant ratio between successive outputs.

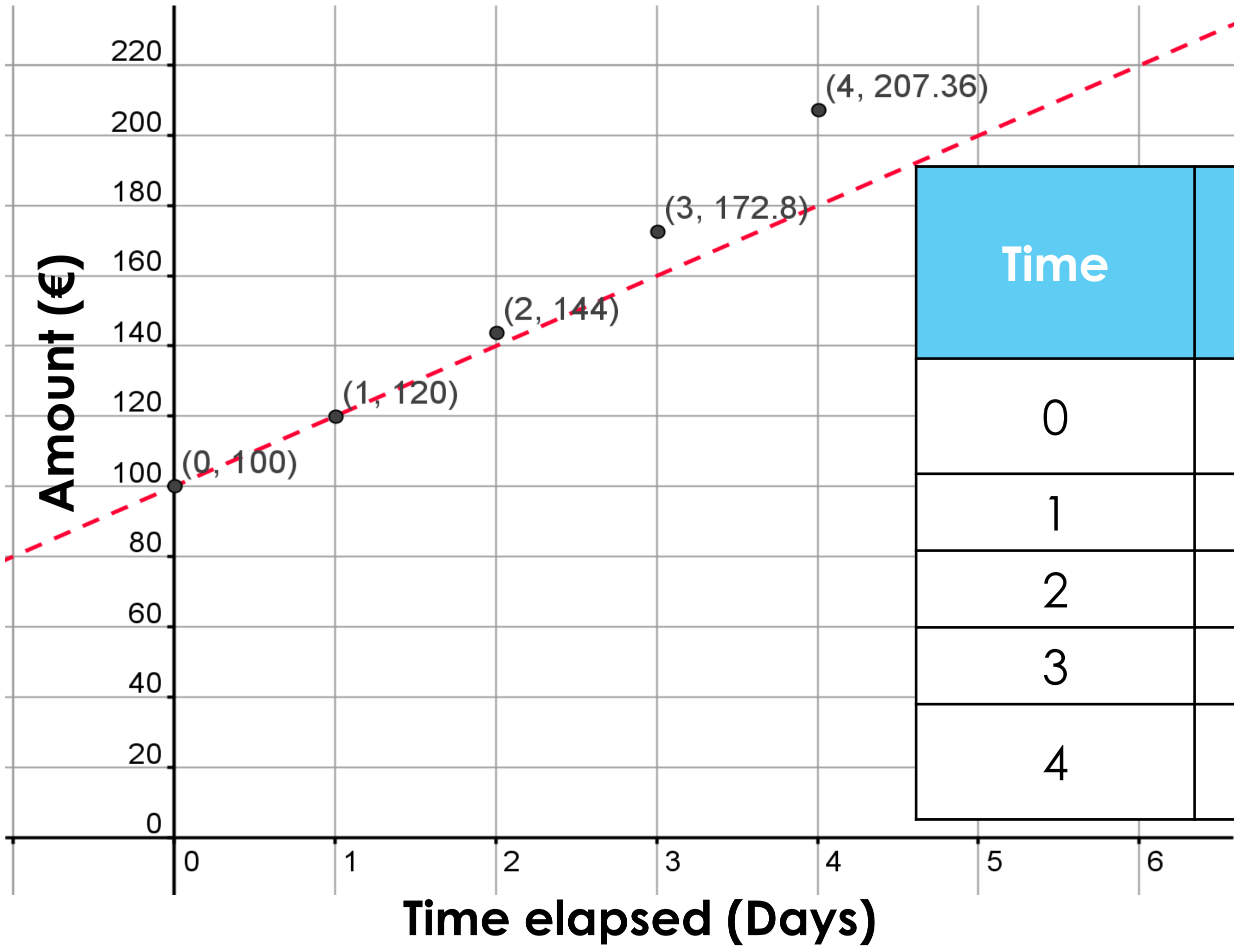
You owe €100. Interest is charged at 20% per day compounded daily.
How much do you owe at the end of 5 days if you do not pay back anything?

Project Maths: Workshop 7



Time	Amount
0	100
1	120
2	144
3	172.80
4	207.36

$$F = P(1.2)^t$$



Time	Amount	Change	Change of Change
0	100		
		20	
1	120		4
		24	
2	144		4.80
		28.80	
3	172.80		5.76
		34.56	
4	207.36		

Compound interest formula

Variables denoting money are in capital letters!

$$F = P(1+i)^t$$

Matamaitic an airgeadais — Financial mathematics

Iontu seo a leanas, is é t an fad ama ina bhlianta agus is é i an ráta bliantúil úis, dímheasa nó fáis, agus é sloinnte mar dheachúil nó mar chodán (ionas go seasann $i = 0.08$ do ráta 8%, mar shampla)*.

In all of the following, t is the time in years and i is annual rate of interest, depreciation or growth, expressed as a decimal or fraction (so that, for example, $i = 0.08$ represents a rate of 8%)*

Ús iolraithe F = luach deiridh, P = príomhshuim	$F = P(1+i)^t$	**Compound interest** F = final value, P = principal
Luach láithreach P = luach láithreach, F = luach deiridh	$P = \dfrac{F}{(1+i)^t}$	**Present value** P = present value, F = final value
Dímheas – modh an chomhardaithe laghdaithigh F = luach déanach, P = luach tosaigh	$F = P(1-i)^t$	**Depreciation – reducing balance method** F = later value, P = initial value
Dímheas – an modh dronlíneach A = méid an dímheasa bhliantúil P = luach tosaigh, S = dramhluach t = saolré eacnamaíoch fhónta	$A = \dfrac{P-S}{t}$	**Depreciation – straight line method** A = annual depreciation amount P = initial value, S = scrap value t = useful economic life

*Bíonn feidhm ag na foirmlí sin freisin nuair a bhítear ag athiolrú i gceann eatraimh chothroma seachas blianta. Sa chás sin, déantar t a thomhas sa tréimhse chuí ama, agus is é i an ráta don tréimhse.

*The formulae also apply when compounding at equal intervals other than years. In such cases, t is measured in the relevant periods of time, and i is the period rate.

Depreciation – Reducing Balance Method

$$F = P(1+i)^t$$

Matamaitic an airgeadais		Financial mathematics
Iontu seo a leanas, is é t an fad ama ina bhlianta agus is é i an ráta bliantúil úis, dímheasa nó fáis, agus é slo… go seasan…		In all of the following, t is the time in years and i is annual rate of interest, depreciation or growth, …t, for
Ús iolrai… F = luach…		**…terest** …ncipal
Luach lái… P = luach…		**…value** …value
Dímheas – modh an chomhardaithe laghdaithigh F = luach déanach, P = luach tosaigh	$F = P(1-i)^t$	**Depreciation – reducing balance method** F = later value, P = initial value
Dímheas – an modh dronlíneach A = méid an dímheasa bhliantúil P = luach tosaigh, S = dramhluach t = saolré eacnamaíoch fhónta	$A = \frac{P-S}{t}$	**Depreciation – straight line method** A = annual depreciation amount P = initial value, S = scrap value t = useful economic life

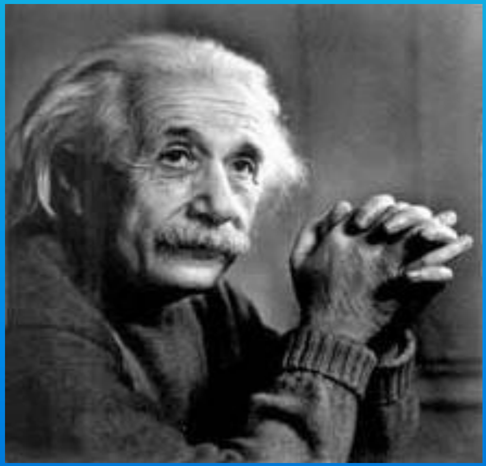
The formulae also apply when compounding at equal intervals other than years.
In such cases, t is measured in the relevant periods of time and i is the period rate

*Bíonn feidhm ag na foirmlí sin freisin nuair a bhítear ag athiolrú i gceann eatraimh chothroma seachas blianta. Sa chás sin, déantar t a thomhas sa tréimhse chuí ama, agus is é i an ráta don tréimhse.

*The formulae also apply when compounding at equal intervals other than years. In such cases, t is measured in the relevant periods of time, and i is the period rate.

"*The most powerful force in the world is compound interest*"

(Albert Einstein)



AER, EAR, CAR
and
interest rates other than annual

A is for annual

Savings and Investments

AER (annual equivalent/effective rate) tells you the interest you would earn if the interest was paid and compounded annually.
(Interest rates are sometimes quoted for periods other than yearly.)

Allows investors to make ***comparisons between savings accounts*** which pay interest at different intervals.

It may or may not include charges.

The financial regulator's office considers the terms AER/EAR and CAR all to be equivalent. The term CAR is approved for use in relation to tracker bonds – for other investment products the regulator considers the acronym AER or EAR should be used. APR is reserved for loans and credit agreements.

A Government Bond Investment

The Government's Nationality Solidarity Bond offers 41.7% net return (after tax) after 10 years. Calculate the AER (after tax) for the bond.

$$F = P(1+i)^t$$

$$141.7 = 100(1+i)^{10}$$

$$\sqrt[10]{1.417} = 1 + i = 1.035468722 \Rightarrow i = 0.0355$$

The AER is 3.55% after tax.

9 Month Fixed Term Converted to AER

A bank has offered a 9 month fixed term reward account paying 2.55% on maturity , for new funds from €10,000 to €500,000.
(You get your money back in 9 months time, along with 2.55% interest.)
Confirm that this is, as advertised, an EAR of 3.41%.

Solution 1 (Assuming 3.41% AER)

$$F = P(1+i)^t$$

Assuming €100 is invested, what will it amount to in $\frac{9}{12}$ years at 3.41% AER?

$$F = 100(1.0341)^{\frac{9}{12}}$$ (matching periodic rate and period)

$$F = €102.55$$

If €100 becomes €102.55 in 9 months, this represents an interest rate for the 9 months of 2.55%.

Solution 2

Alternatively, calculate the AER given that the interest rate for 9 months is 2.55%

$$F = P(1+i)^t$$

$$1.0255 = (1+i)^{9/12}$$

$$(1.0255)^{12/9} = 1+i$$

$i = 1.0341 \Rightarrow$ AER = annual equivalent rate = 3.41%

NTMA Brochure 1
16 December 2012

Ireland™ State Savings
NATIONAL TREASURY MANAGEMENT AGENCY

The main NTMA State Savings™ products are:

Actual BEFORE DIRT		NTMA State Savings™ Products	Actual AFTER DIRT	
Total Return	Gross AER[1]		Net AER[1]	Total Return
	0.25%	Ordinary Deposit Account (DEMAND)[5]	0.17%	
	1.00%	Deposit Account Plus (30 DAY NOTICE)[5]	0.67%	
		Prize Bonds ([illegible])[6]		
7%	2.28%	3 year Savings Bonds[2]	2.28%	7.00%
12%	2.87%	4 year National Solidarity Bonds[3]	2.57%	10.68%
15%	2.83%	5 year Savings Certificates[2]	2.83%	15.00%
17%	2.90%	6 year Instalment Savings[2,4] (ALSO FOR CHILD BENEFIT)	2.90%	17.00%
45%	3.79%	10 Year National Solidarity Bonds[3]	3.55%	41.70%

[1] *AER = "Annual Equivalent Rate"*
[2] *Tax Free – Not subject to tax in Ireland.*
[3] *Partial Tax – Annual Interest of 1% is subject to DIRT (33% Jan 2013) – bonus is tax free. AER calculated on the assumption that the recipient of the annual NSB interest does not reinvest it.*
[4] *Save in 12 monthly instalments (Max. €1,000 per month) and leave for 5 years. AER calculation assumes a 5½ year average life.*
[5] *Subject to DIRT = Deposit Interest Retention Tax (33% from January 2013).*
[6] *Prize Bonds - Prize Fund is 2.25% pa. One €1 MILLION prize awarded each month, 1,000's of other weekly cash prizes (tax free in Ireland).*

See our range of NTMA State Savings™ Brochures:

1. A Guide to NTMA State Savings™
2. NTMA State Savings™ - Summary of Products
3. The National Solidarity Bond
4. Prize Bonds

For more information on NTMA State Savings™

Web: www.StateSavings.ie
Telephone: 1850 30 50 60
SMS Text: 0852 30 50 60
E-mail: Service@StateSavings.ie
Visit: Any Post Office
Mail to: State Savings, GPO, FREEPOST, Dublin 1

An Post and the Prize Bond Company are agents of the National Treasury Management Agency

State Savings™ products are subject to Terms and Conditions.

NTMA Brochure 1
16 December 2012

Ireland™ State Savings
NATIONAL TREASURY MANAGEMENT AGENCY

A Guide to NTMA State Savings™

POST

NTMA Brochure 1
16 December 2012

An Post and The Prize Bond Company are agents of the National Treasury Management Agency

Question 1

Using one of the brochures for NTMA State Savings products, as shown below, verify the Gross AER values, given the total return.

$$107 = 100(1+i)^3$$
$$(1.07)^{1/3} = 1+i$$
$$0.0228 = i$$
$$2.28\% = r$$

$$115 = 100(1+i)^5$$
$$(1.15)^{1/5} = 1+i$$
$$0.02834 = i$$
$$2.83\% = r$$

$$145 = 100(1+i)^{10}$$
$$(1.45)^{1/10} = 1+i$$
$$0.037855 = i$$
$$3.79\% = r$$

The main NTMA State Savings™ products are:

Actual BEFORE DIRT		NTMA State Savings™ Products	Actual AFTER DIRT	
Total Return	Gross AER[1]		Net AER[1]	Total Return
	0.25%	Ordinary Deposit Account (DEMAND)[5]	0.17%	
	1.00%	Deposit Account Plus (30 DAY NOTICE)[5]	0.67%	
		Prize Bonds (WEEKLY DRAW)[6]		
7%	2.28%	3 year Savings Bonds[2]	2.28%	7.00%
12%	2.87%	4 year National Solidarity Bonds[3]	2.57%	10.68%
15%	2.83%	5 year Savings Certificates[2]	2.83%	15.00%
17%	2.90%	6 year Instalment Savings[2,4] (ALSO FOR CHILD BENEFIT)	2.90%	17.00%
45%	3.79%	10 Year National Solidarity Bonds[3]	3.55%	41.70%

1 AER = "Annual Equivalent Rate"
2 Tax Free – Not subject to tax in Ireland.
3 Partial Tax – Annual interest of 1% is subject to DIRT (33% Jan 2013) – bonus is tax free. AER calculated on the assumption that the recipient of the annual NSB interest does not reinvest it.
4 Save in 12 monthly instalments (Max. €1,000 per month) and leave for 5 years. AER calculation assumes a 5¼ year average life.
5 Subject to DIRT = Deposit Interest Retention Tax (33% from January 2013).
6 Prize Bonds - Prize Fund is 2.25% pa. One €1 MILLION prize awarded each month, 1,000's of other weekly cash prizes (tax free in Ireland).

Question 2

See Question Booklet

$$0.01\times 4$$
$$=0.04\times 67\%$$
$$=0.0268$$
$$2.68\%=r$$

$$=2.68\%+8\%$$
$$=10.68\%$$

$$110.68=100(1+i)^4$$
$$(1.1068)^{1/4}=1+i$$
$$0.0257=i$$
$$2.57\%=r$$

$$0.01\times 10$$
$$=0.1\times 67\%$$
$$=0.067$$
$$6.7\%=r$$

$$=6.70\%+35\%$$
$$=41.7\%$$

$$141.70=100(1+i)^{10}$$
$$(1.417)^{1/10}=1+i$$
$$0.03546=i$$
$$3.55\%=r$$

3. National Solidarity Bond 4 year

(2nd Issue)

4 Year National Solidarity Bond

Gross before DIRT	New 4 Year National Solidarity Bond	Net after DIRT
4.00%	Annual 1% interest payments over 4 years	2.68%
8.00%	The tax free bonus at the end of 4 years	8.00%
12.00%	Total return	10.68%
2.87%	The AER (Annual Equivalent Rate) is	2.57%

Minimum Savings - €100
Maximum Savings - €250,000 per individual

4. National Solidarity Bond 10 year

(2nd Issue)

10 Year National Solidarity Bond

Gross before DIRT	10 Year National Solidarity Bond	Net after DIRT
10.00%	Annual 1% interest payments over 10 years	6.70%
35.00%	The tax free bonus at the end of 10 years	35.00%
45.00%	Total return	41.70%
3.79%	The AER (Annual Equivalent Rate) is	3.55%

- *The gross 1% annual interest is subject to the prevailing DIRT rate (33% in Jan 2013) - NET 0.67% p.a.*
- *AER calculated on assumption that the saver does not reinvest their annual 1% interest payment.*

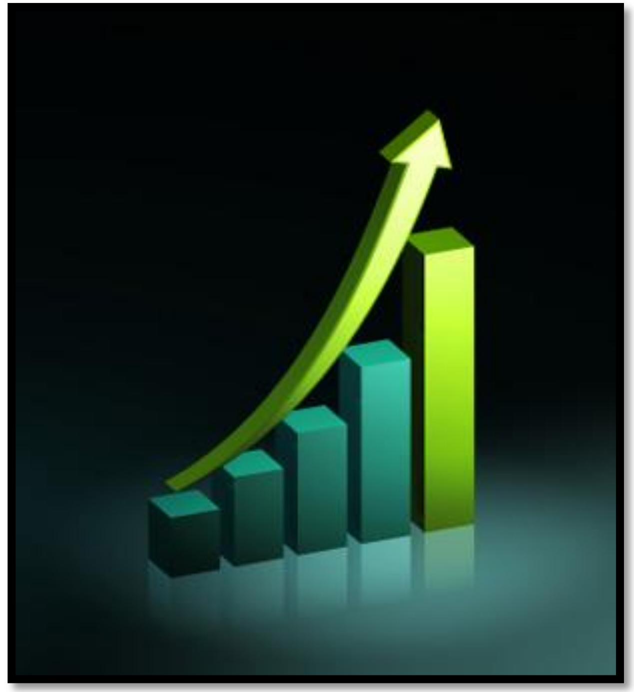
Minimum Savings - €100
Maximum Savings - €250,000 per individual

Annuities

An annuity is a ***sequence of*** periodic equal contributions made by or to an individual for a specified term.

Examples:

- Regular deposits in a savings account
- Periodic payments to a retired person from a pension fund or lotto winnings (USA)
- Loans are usually paid off by an annuity



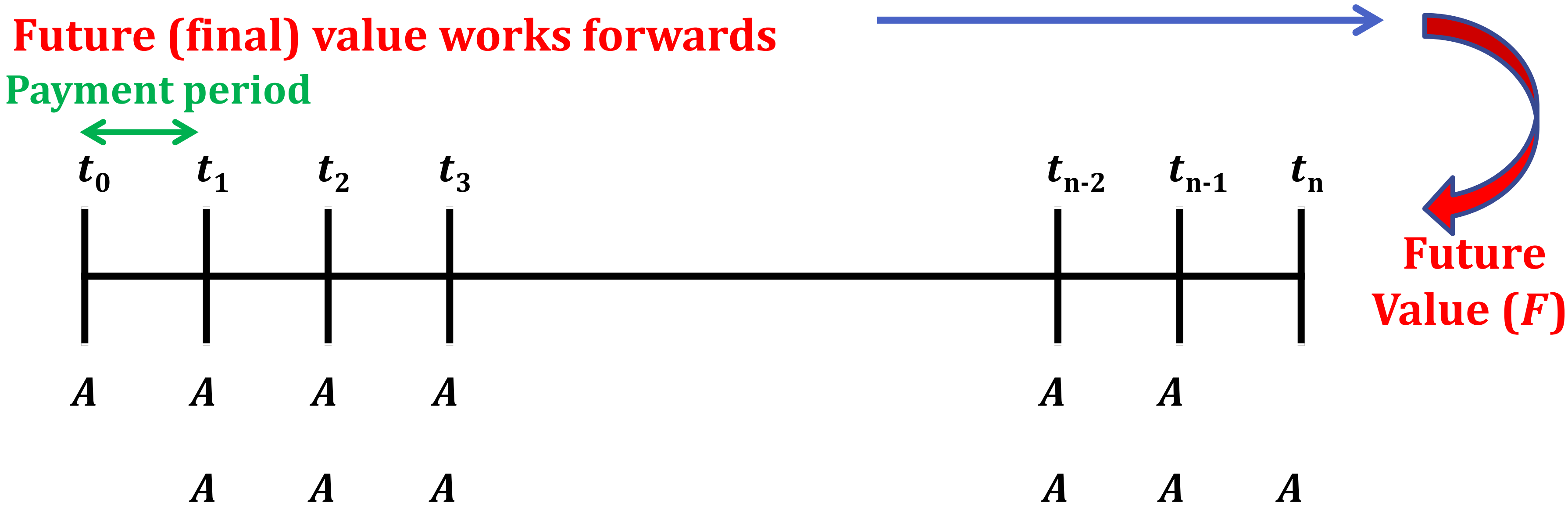
The Time Value of Money

Present value ⇄ Future (Final) value

Annuities Involving Future Values

Regular saver accounts

n regular fixed payments, of amount €A, over a specified period of time



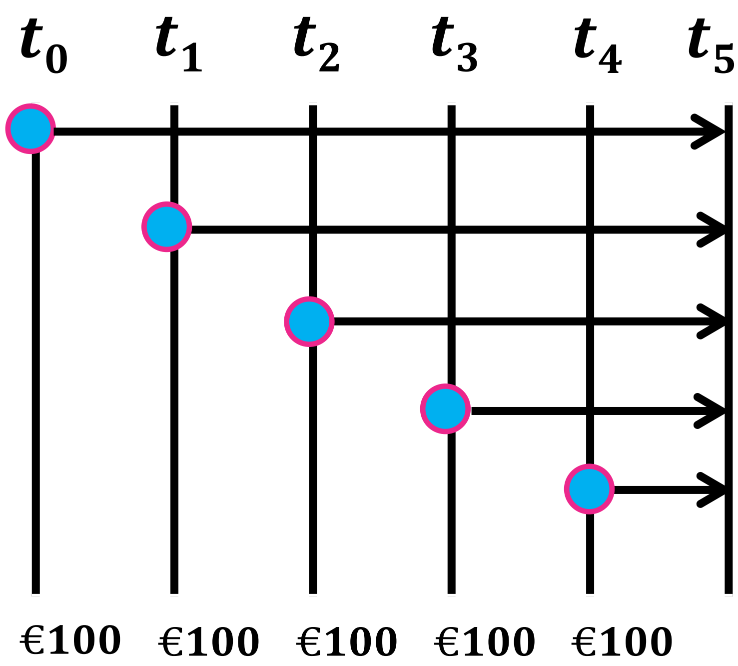
The payment A can be paid either

- at the beginning or
- at the end of the payment period.

If we are **investing money** into an account, we will be working with the **future values** of the payments into the account, since we are **saving money for the future**.

Question 3 [Future Value – Annuity Due]

Five payments, each of €100, are paid into an account at regular intervals, at the beginning of each of five years, at an AER of 3%, paid an compounded annually.



$$F_1 = 100(1.03)^5$$
$$F_2 = 100(1.03)^4$$
$$F_3 = 100(1.03)^3$$
$$F_4 = 100(1.03)^2$$
$$F_5 = 100(1.03)^1$$

Your Turn **Q3 (a)** and **(b)**

What is the future value of the annuity?

$$\sum_{r=1}^{n} F_r = \text{The sum of all the separate future values} = \text{The future value of the annuity}$$

$$\sum_{r=1}^{n} F_r = 100(1.03)^1 + 100(1.03)^2 + 100(1.03)^3 + 100(1.03)^4 + 100(1.03)^5$$

$$\sum_{r=1}^{n} F_r = 100(1.03)^1 + 100(1.03)^2 + 100(1.03)^3 + 100(1.03)^4 + 100(1.03)^5$$

This is a geometric series with $a = 100(1.03)$ and $r = 1.03$.

$$S_5 = \frac{a(r^n - 1)}{(r-1)}$$

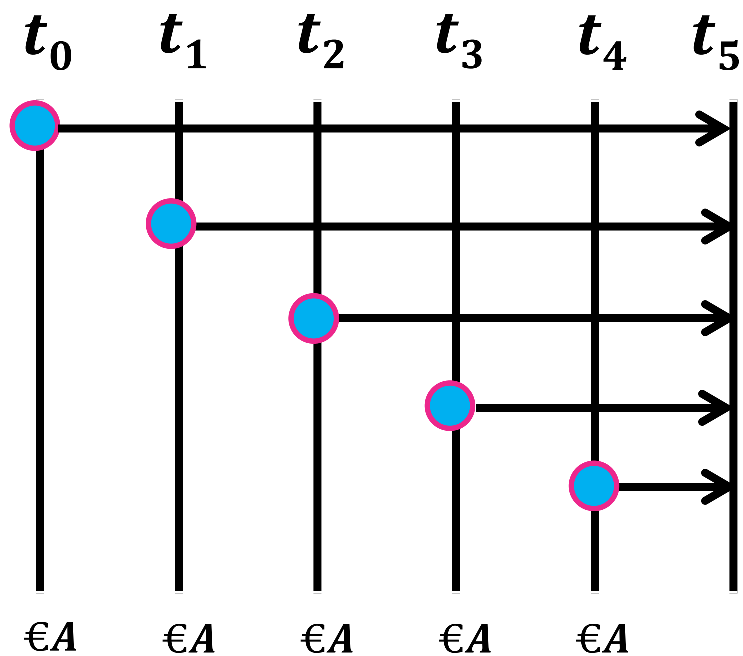
$$= \frac{100(1.03)(1.03^5 - 1)}{(1.03 - 1)}$$

$$= \frac{100(1.03)(1.03^5 - 1)}{0.03}$$

$$= €546.84$$

Question 3 (d) [Future Value – Annuity Due]

Generalise the above procedure, to find the future value of five payments of €A , paid into an account at regular yearly intervals, at the beginning of each of five years, where i is the AER expressed as a decimal, and the interest is paid and compounded annually.



$$F_1 = A(1+i)^5$$
$$F_2 = A(1+i)^4$$
$$F_3 = A(1+i)^3$$
$$F_4 = A(1+i)^2$$
$$F_5 = A(1+i)^1$$

What is the future value of the annuity?

$$\sum_{r=1}^{n} F_r = \text{The sum of all the separate future values} = \text{The future value of the annuity}$$

$$\sum_{r=1}^{n} F_r = A(1+i)^1 + A(1+i)^2 + A(1+i)^3 + A(1+i)^4 + A(1+i)^5$$

$$\sum_{r=1}^{n} F_r = A(1+i)^1 + A(1+i)^2 + A(1+i)^3 + A(1+i)^4 + A(1+i)^5$$

This is a geometric series with $a = A(1+i)$ and $r = 1+i$.

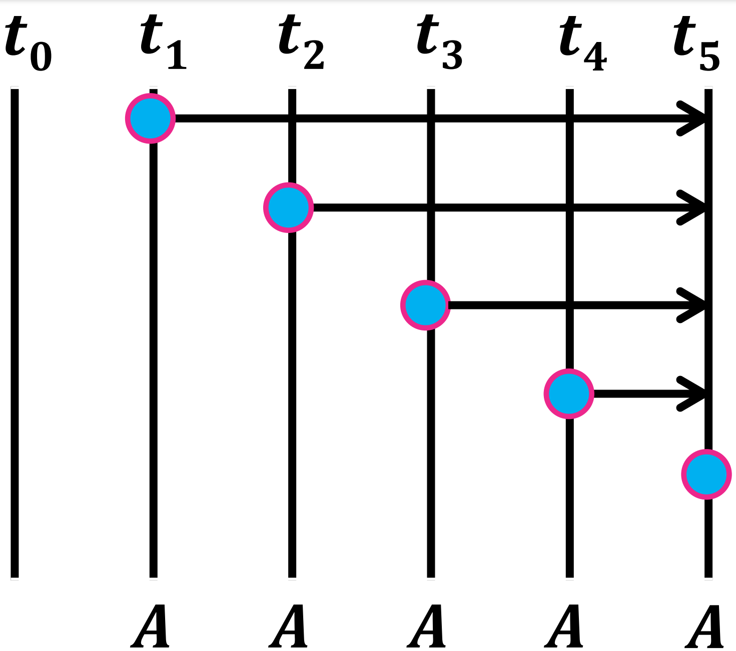
$$S_5 = \frac{a(r^n - 1)}{(r-1)}$$

$$= \frac{A(1+i)((1+i)^5 - 1)}{(1+i-1)}$$

$$= \frac{A(1+i)((1+i)^5 - 1)}{i}$$

Question 4 [Future Value – Ordinary Annuity and Comparison with Annuity due)

What is the future value of five payments, each of €A, paid into an account at regular yearly intervals, <u>at the end</u> of each of five years, where i is the AER expressed as a decimal?
(Draw and fill in, a timeline and a table for the situation, as in Q3. above.)



$$F_1 = A(1+i)^4$$
$$F_2 = A(1+i)^3$$
$$F_3 = A(1+i)^2$$
$$F_4 = A(1+i)^1$$
$$F_5 = A$$

Your Turn Q4

What is the future value of the annuity?

$$\sum_{r=1}^{n} F_r = \text{The sum of all the separate future values} = \text{The future value of the annuity}$$

$$\sum_{r=1}^{n} F_r = A + A(1+i)^1 + A(1+i)^2 + A(1+i)^3 + A(1+i)^4$$

$$\sum_{r=1}^{n} F_r = A + A(1+i)^1 + A(1+i)^2 + A(1+i)^3 + A(1+i)^4$$

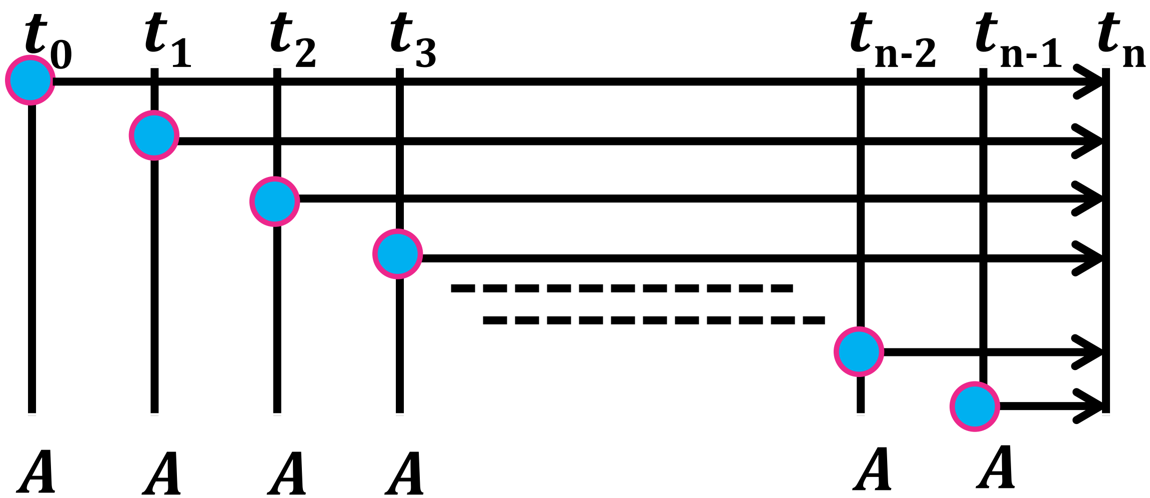
This is a geometric series with $a = A$ and $r = 1 + i$.

$$S_5 = \frac{a(r^n - 1)}{(r-1)}$$

$$= \frac{A((1+i)^5 - 1)}{(1+i-1)}$$

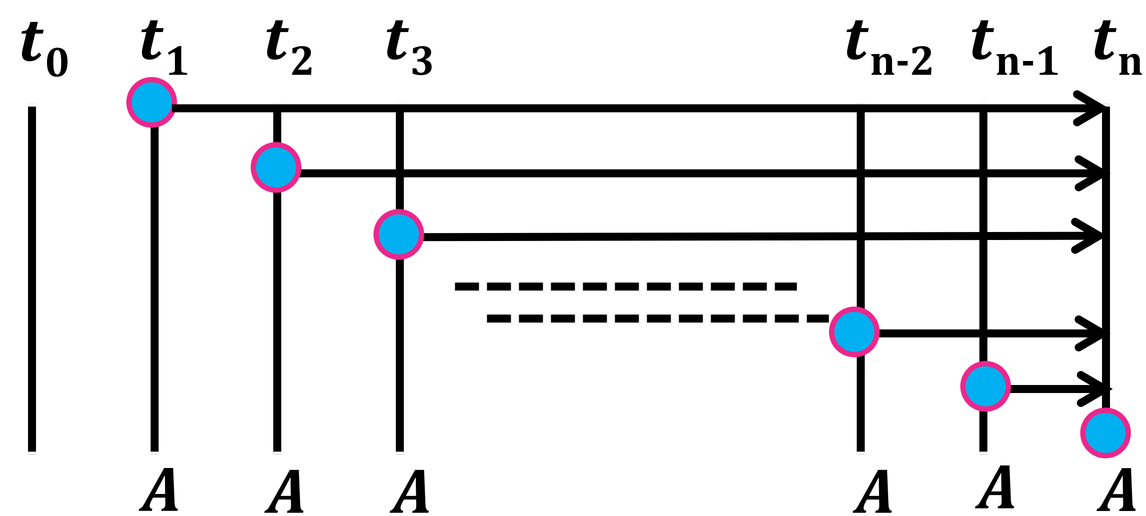
$$= \frac{A((1+i)^5 - 1)}{i}$$

First payment immediately;
last payment at the beginning of the last payment period



$$S_n = \frac{A(1+i)((1+i)^n - 1)}{i}$$

First payment made at the end of the first payment period;
last payment at the end of the last payment period



$$S_n = \frac{A((1+i)^n - 1)}{i}$$

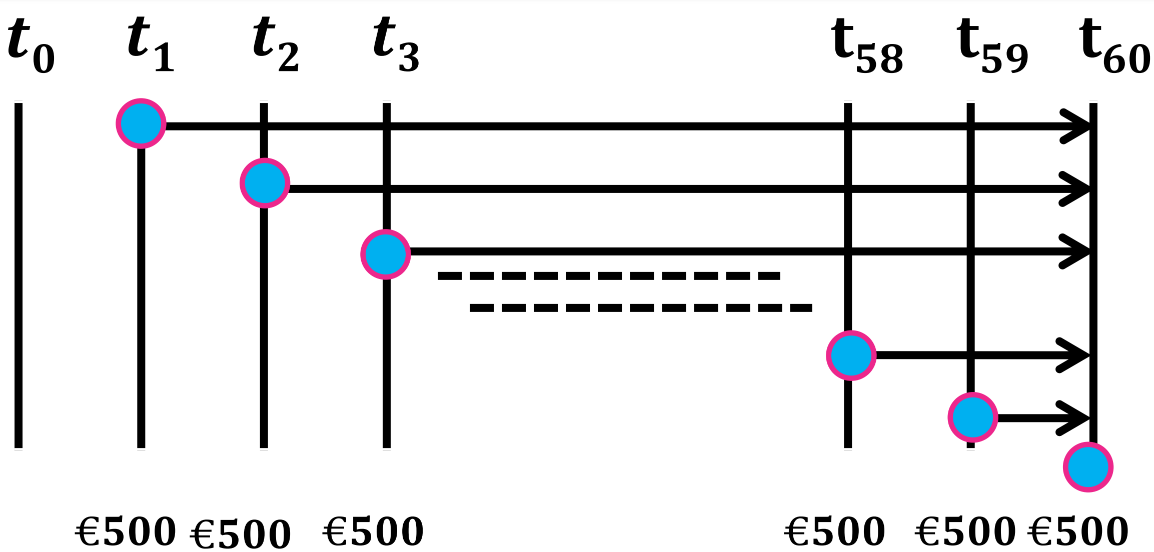
Question 5 [Different Compounding Periods]

Calculate the number of compounding periods in the given time intervals, as the length of the compounding period varies.

Time intervals/years	**Number of Compounding Periods**			
	Interest added every 6 months	Interest added every 3 months	Interest added every month	Interest added every day*
3.5	7	14	42	1277.50
10	20	40	120	3650
40	80	160	480	14600

Question 6 Regular Savings (Future value of an annuity)

At the end of each month a deposit of €500 is made into an account that pays an AER of 8% compounded monthly.
Calculate the final amount (future value) after 5 years?

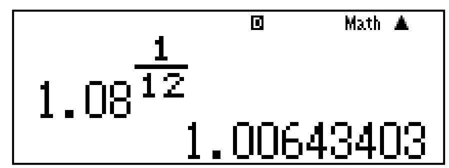


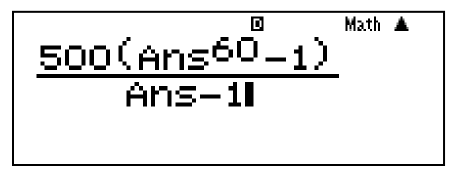
$$F_1 = 500\left(1.08^{1/12}\right)^{59}$$
$$F_2 = 500\left(1.08^{1/12}\right)^{58}$$
$$F_3 = 500\left(1.08^{1/12}\right)^{57}$$
$$\vdots \qquad \vdots$$
$$F_{58} = 500\left(1.08^{1/12}\right)^{2}$$
$$F_{59} = 500\left(1.08^{1/12}\right)^{1}$$
$$F_{60} = €500$$

$$S_{60} = 500 + 500\left(1.08^{1/12}\right) + 500\left(1.08^{1/12}\right)^2 + \cdots + 500\left(1.08^{1/12}\right)^{57} + 500\left(1.08^{1/12}\right)^{58} + 500\left(1.08^{1/12}\right)^{59}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ with } a = 500, r = (1.08)^{1/12} \text{ and } n = 60$$

$$S_{60} = \frac{500\left[\left((1.08)^{\frac{1}{12}}\right)^{60} - 1\right]}{1.08^{\frac{1}{12}} - 1} = €36{,}472.33$$



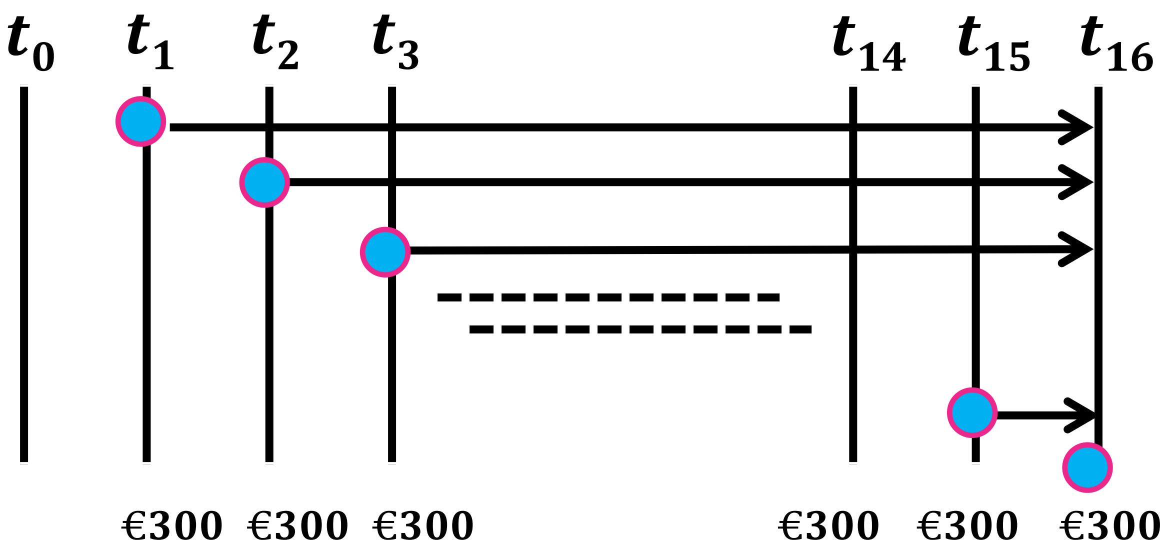


Question 7 [Regular Savings – Future value of an annuity plus lump sum compounding]

Sonya deposits €300 at the end of each quarter in her savings account. The money earns 5.75% (EAR).

(a) How much will this investment be worth at the end of 4 years?

(b) After four years she is offered 6% AER, paid and compounded annually, if she does not withdraw the money for the following two years. How much will her investment amount to at the end of the six years?



$$F_1 = €300\left(1.0575^{1/4}\right)^{15}$$

$$F_2 = €300\left(1.0575^{1/4}\right)^{14}$$

$$F_3 = €300\left(1.0575^{1/4}\right)^{13}$$

$$\vdots \qquad \vdots$$

$$F_{15} = €300\left(1.0575^{1/4}\right)^{1}$$

$$F_{16} = €300$$

Question 7 [Regular Savings – Future value of an annuity plus lump sum compounding]

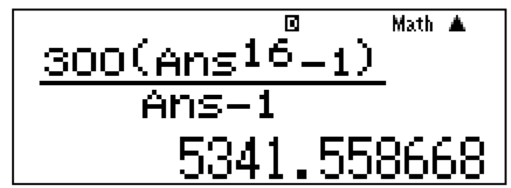
Sonya deposits €300 at the end of each quarter in her savings account. The money earns 5.75% (EAR).

(a) How much will this investment be worth at the end of 4 years?

(b) After four years she is offered 6% AER, paid and compounded annually, if she does not withdraw the money for the following two years. How much will her investment amount to at the end of the six years?

(a) $S_{16} = 300 + 300(1.0575^{1/4})^{1} + \cdots + 300(1.0575^{1/4})^{13} + 300(1.0575^{1/4})^{14} + 300(1.0575^{1/4})^{15}$

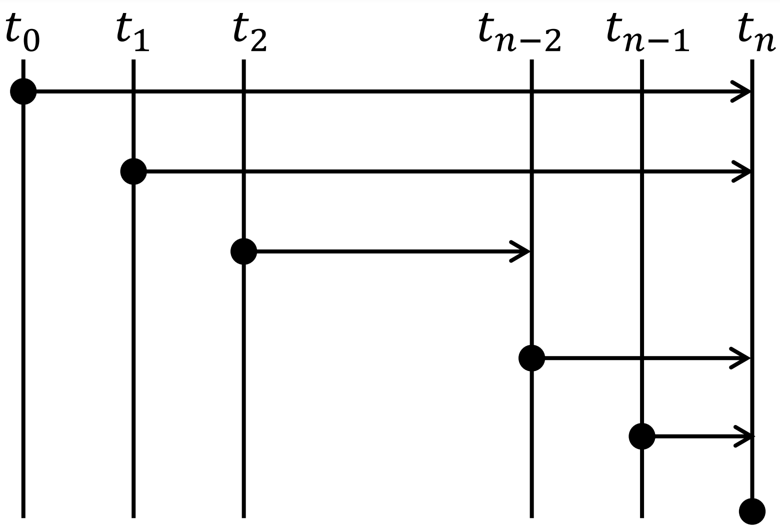
$$S_{16} = \frac{300\left[(1.0575^{1/4})^{16} - 1\right]}{1.0575^{1/4} - 1} = €5{,}341.56$$



(b) $F = 5341.56(1.06)^{2} = €6001.78$

Question 8 [Regular Savings – Future Value of an annuity]

How long should an investor continue to make monthly investments of €1000 at a rate of 3% AER if she wishes to have at least €100,000 in a savings account. Assume that the first payment is made immediately and that the last payment is made on the day that the investment matures.



$$F_1 = 1000(1.03^{1/12})^n$$

$$F_2 = 1000(1.03^{1/12})^{n-1}$$

$$F_3 = 1000(1.03^{1/12})^{n-2}$$

$$F_{n-1} = 1000(1.03^{1/12})^2$$

$$F_n = 1000(1.03^{1/12})^1$$

$$F_{n+1} = 1000$$

Question 8 [Regular Savings – Future Value of an annuity]

How long should an investor continue to make monthly investments of €1000 at a rate of 3% AER if she wishes to have at least €100,000 in a savings account. Assume that the first payment is made immediately and that the last payment is made on the day that the investment matures.

$$100{,}000 = \underbrace{1000 + 1000\left(1.03^{1/12}\right) + \cdots + 1000\left(1.03^{1/12}\right)^{n-2} + 1000\left(1.03^{1/12}\right)^{n-1} + 1000\left(1.03^{1/12}\right)^{n}}_{a=1000,\ r=1.03^{1/12},\ n=n+1}$$

$$100000 = \frac{1000\left[\left(1.03^{1/12}\right)^{n+1} - 1\right]}{1.03^{1/12} - 1}$$

$$100\left(1.03^{1/12} - 1\right) = \left(1.03^{1/12}\right)^{n+1} - 1$$

$$100\left(1.03^{1/12} - 1\right) + 1 = \left(1.03^{1/12}\right)^{n+1}$$

$$\log\left[100\left(1.03^{1/12} - 1\right) + 1\right] = (n+1)\log 1.03^{1/12}$$

$$\frac{\log\left[100\left(1.03^{1/12} - 1\right) + 1\right]}{\log 1.03^{1/12}} = n + 1$$

$$89.49 = n + 1$$

$$88.49 = n$$

```
log(100(Ans-1)+
   log(Ans)
            89.49272735
```

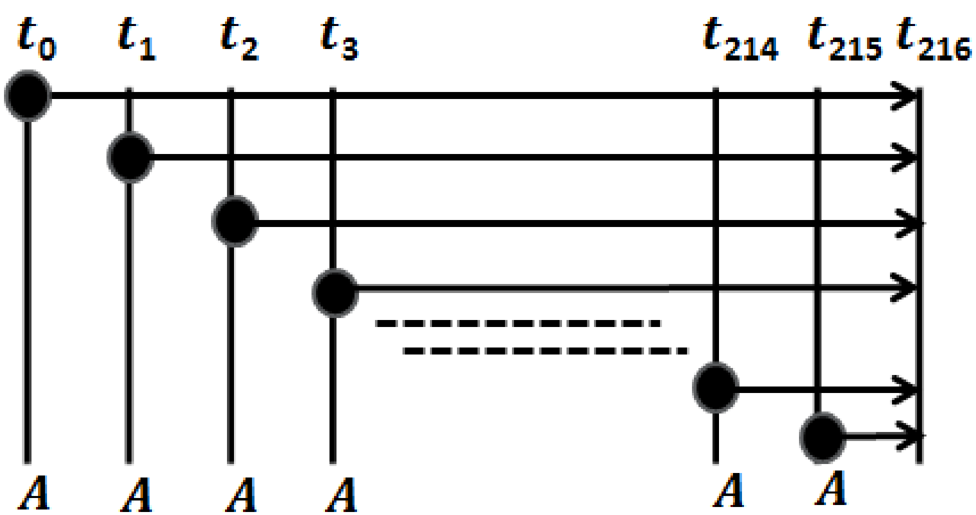
To have at least €100,000 we need 89 months which is 7 years 5 months.

Question 9 [Saving for College Fees]

Ben wants to save up some money so that he can have €40 000 saved for his unborn daughter on her 18th birthday to help with college fees. On the day his daughter is born he begins making equal monthly payments into an account that pays 3.5% AER which is paid and compounded monthly.

His last payment into the account is due one month before his daughter turns 18.

Calculate monthly payment required to achieve the fund of €40 000 in the time given.



$$F_1 = A\left(1.035^{1/12}\right)^{216}$$

$$F_2 = A\left(1.035^{1/12}\right)^{215}$$

$$F_3 = A\left(1.035^{1/12}\right)^{214}$$

$$F_3 = A\left(1.035^{1/12}\right)^{213}$$

$$\vdots \qquad \vdots$$

$$F_{215} = A\left(1.035^{1/12}\right)^{2}$$

$$F_{216} = A\left(1.035^{1/12}\right)^{1}$$

Question 9 [Saving for College Fees]

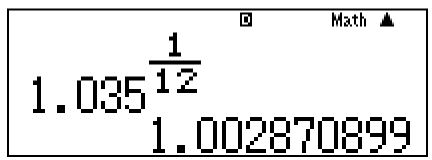
Ben wants to save up some money so that he can have €40 000 saved for his unborn daughter on her 18th birthday to help with college fees. On the day his daughter is born he begins making equal monthly payments into an account that pays 3.5% AER which is paid and compounded monthly.

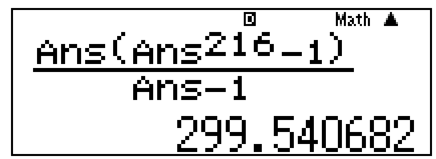
His last payment into the account is due one month before his daughter turns 18.

Calculate monthly payment required to achieve the fund of €40 000 in the time given.

$$S_{216} = 40000 = A\left(1.035^{1/12}\right) + A\left(1.035^{1/12}\right)^{2} + \cdots + A\left(1.035^{1/12}\right)^{215} + A\left(1.035^{1/12}\right)^{216}$$

$$40000 = \frac{A\left(1.035^{1/12}\right)\left[\left(1.035^{1/12}\right)^{216} - 1\right]}{1.035^{1/12} - 1}$$





$$40000 = 299.540682A$$

$$€133.54 = A$$

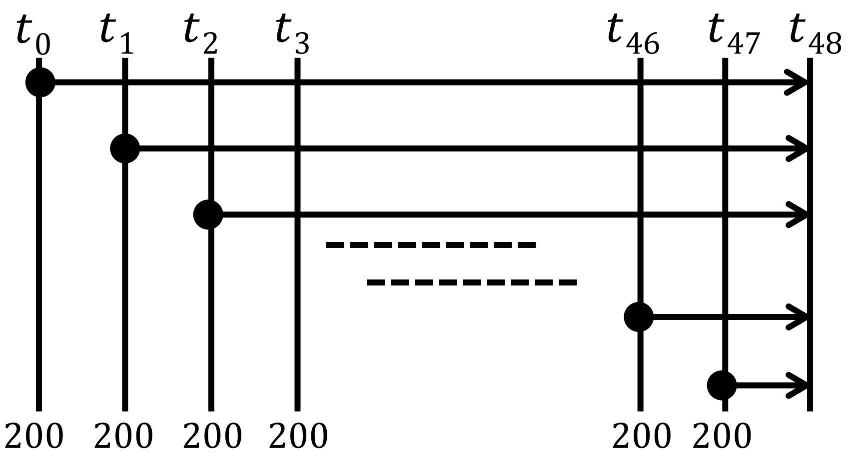
What if he waits until her 5th birthday to start investing for her college fees?

$$40000 = \frac{A\left(1.035^{1/12}\right)\left[\left(1.035^{1/12}\right)^{156} - 1\right]}{1.035^{1/12} - 1}$$

A= €203.04

Ailish deposits €200 at the beginning of each month for four years, into a savings account which pays 2.5% AER paid and compounded monthly. At the end of each year she also deposits €1000 into this account.
How much money will she have in the account at the end of the 4 years?

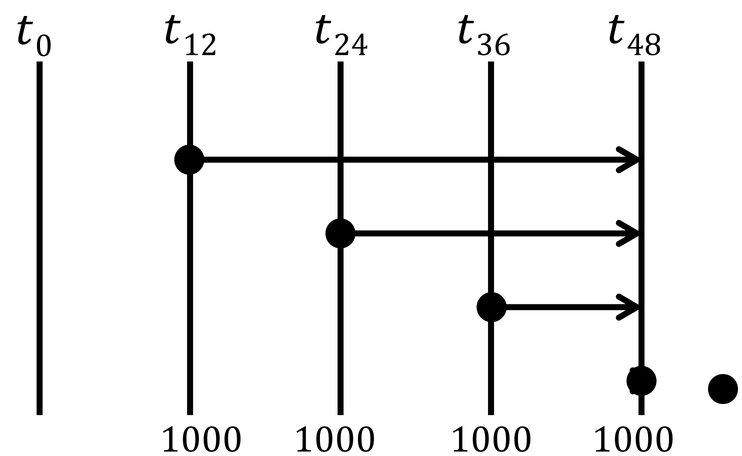
Annuity 1



$$S_{48} = \frac{200(1.025)^{\frac{1}{12}}\left[\left(1.025^{\frac{1}{12}}\right)^{48} - 1\right]}{(1.025)^{\frac{1}{12}} - 1}$$

$$= €10\,100.48$$

Annuity 2



$$S_4 = \frac{1{,}000(1.025^4 - 1)}{1.025 - 1}$$

$$= €4152.52$$

Total value of the annuities = €14 253.00

Question 11 [Sinking Fund]

Companies often purchase equipment and use it for a specified time period. The old equipment is then sold at scrap value and new, upgraded equipment is bought. In order to finance the purchasing of the new equipment, the company sets up, in advance, an annuity called a sinking fund.

A university buys a bus that costs €105,000 and its useful life is 5 years. It depreciates at 12% p.a. reducing balance. Assume that the cost of this type of bus increases at 3% AER, compounded annually.

The old bus will be sold at scrap value in 5 years and the proceeds will be used together with a sinking fund to buy a new bus. The university will make payments each month into a savings account giving a 4.4% AER compounded monthly. The first payment will be made at the end of the first month and the last payment will be made at the end of the 5 year period.

(a) What is the scrap value of the bus after 5 years?

(b) Calculate the cost of the a new bus of the same type, in 5 years' time.

(c) Calculate the amount required in the sinking fund.

(d) Calculate the monthly repayments required for the purchase of the new bus.

We need to separate information so we don't mix up some rates and some periods.

Question 11 [Sinking Fund]

Companies often purchase equipment and use it for a specified time period. The old equipment is then sold at scrap value and new, upgraded equipment is bought. In order to finance the purchasing of the new equipment, the company sets up, in advance, an annuity called a sinking fund.

A university buys a bus that costs €105 000 and its useful life is 5 years. It depreciates at 12% p.a. reducing balance. Assume that the cost of this type of bus increases at 3% AER, compounded annually.

The old bus will be sold at scrap value in 5 years and the proceeds will be used together with a sinking fund to buy a new bus. The university will make payments each month into a savings account giving a 4.4% AER compounded monthly. The first payment will be made at the end of the first month and the last payment will be made at the end of the 5 year period.

(a) What is the scrap value of the bus after 5 years?

(a)

$$F_o = 105\,000(1-0.12)^5$$

$$= €55\,411.85$$

Question 11 [Sinking Fund]

Companies often purchase equipment and use it for a specified time period. The old equipment is then sold at scrap value and new, upgraded equipment is bought. In order to finance the purchasing of the new equipment, the company sets up, in advance, an annuity called a sinking fund.

A university buys a bus that costs €105 000 and its useful life is 5 years. It depreciates at 12% p.a. reducing balance. Assume that the cost of this type of bus increases at 3% AER, compounded annually.

The old bus will be sold at scrap value in 5 years and the proceeds will be used together with a sinking fund to buy a new bus. The university will make payments each month into a savings account giving a 4.4% AER compounded monthly. The first payment will be made at the end of the first month and the last payment will be made at the end of the 5 year period.

(b) Calculate the cost of the a new bus of the same type, in 5 years' time

(b)

$$F_n = 105\,000(1+0.03)^5$$
$$= €121\,723.78$$

Question 11 [Sinking Fund]

Companies often purchase equipment and use it for a specified time period. The old equipment is then sold at scrap value and new, upgraded equipment is bought. In order to finance the purchasing of the new equipment, the company sets up, in advance, an annuity called a sinking fund.

A university buys a bus that costs €105 000 and its useful life is 5 years. It depreciates at 12% p.a. reducing balance. Assume that the cost of this type of bus increases at 3% AER, compounded annually.

The old bus will be sold at scrap value in 5 years and the proceeds will be used together with a sinking fund to buy a new bus. The university will make payments each month into a savings account giving a 4.4% AER compounded monthly. The first payment will be made at the end of the first month and the last payment will be made at the end of the 5 year period.

(c) Calculate the amount required in the sinking fund.

(c) Sinking Fund $= F_n - F_o$

$$= €121\,723.78 - €55\,411.85$$

$$= €66\,311.93$$

Question 11 [Sinking Fund]

Companies often purchase equipment and use it for a specified time period. The old equipment is then sold at scrap value and new, upgraded equipment is bought. In order to finance the purchasing of the new equipment, the company sets up, in advance, an annuity called a sinking fund.

A university buys a bus that costs €105 000 and its useful life is 5 years. It depreciates at 12% p.a. reducing balance. Assume that the cost of this type of bus increases at 3% AER, compounded annually.

The old bus will be sold at scrap value in 5 years and the proceeds will be used together with a sinking fund to buy a new bus. The university will make payments each month into a savings account giving a 4.4% AER compounded monthly. The first payment will be made at the end of the first month and the last payment will be made at the end of the 5 year period.

(d) Calculate the monthly repayments required for the purchase of the new bus.

(d)

$$66\,311.93 = A + A\left(1.044^{1/12}\right)^{1} + \cdots + A\left(1.044^{1/12}\right)^{57} + A\left(1.044^{1/12}\right)^{58} + A\left(1.044^{1/12}\right)^{59}$$

Geometric series with $a = A$, $r = 1.044^{1/12}$ and $n = 60$

$$66\,311.93 = \frac{A\left[\left(1.044^{1/12}\right)^{60} - 1\right]}{1.044^{1/12} - 1}$$

$$66\,3311.93 = 66.82847306A$$

$$\Rightarrow €992.27 = A$$