Module 4 – Solutions

4.1 (a) Yes
(b) No
(c) Yes
(d) Yes
(e) No

4.2

(a) $P(B) = \frac{60}{100} = \frac{3}{5} = 0.6$

(b) $P(\text{Boy not wearing glasses}) = \frac{24}{100} = \frac{6}{25} = 0.24$

(c) $B: \text{Boy}$ $NG: \text{Not wearing glasses}$

Contingency table: $P(B \mid NG) = \frac{24}{50} = \frac{12}{25} = 0.48$

Conditional probability rule: $P(B \mid NG) = \frac{P(B \text{ and } NG)}{P(NG)} = \frac{24/100}{50/100} = \frac{12}{25} = 0.48$

(d) $B: \text{Boy}$ $NG: \text{Not wearing glasses}$

Contingency table: $P(NG \mid B) = \frac{24}{60} = \frac{2}{5} = 0.4$

Conditional probability rule: $P(NG \mid B) = \frac{P(NG \text{ and } B)}{P(B)} = \frac{24/100}{60/100} = \frac{2}{5} = 0.4$

(e) $B: \text{Boy}$ $NG: \text{Not wearing glasses}$

$P(NG \mid B) = 0.4$

$P(B \mid NG) = 0.48$

$\Rightarrow P(NG \mid B) \neq P(B \mid NG)$

(f) $G: \text{Girl}$ $GL: \text{Wearing glasses}$

Contingency table: $P(G \mid GL) = \frac{14}{50} = \frac{7}{25} = 0.28$

Conditional probability rule: $P(G \mid GL) = \frac{P(G \text{ and } GL)}{P(GL)} = \frac{14/100}{50/100} = \frac{7}{25} = 0.28$

(g) $G: \text{Girl}$ $GL: \text{Wearing glasses}$

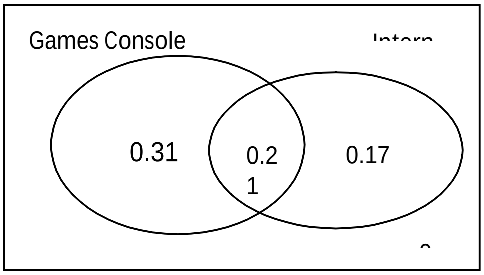
Contingency table: $P(GL \mid G) = \frac{14}{40} = \frac{7}{20} = 0.35$

Conditional probability rule: $P(GL \mid G) = \frac{P(GL \text{ and } G)}{P(G)} = \frac{14/100}{40/100} = \frac{7}{20} = 0.35$

$P(G \mid GL) = \frac{14}{50} = \frac{7}{50} = 0.28$ [From part (f)]

$\Rightarrow P(G \mid GL) \neq P(GL \mid G)$

4.3

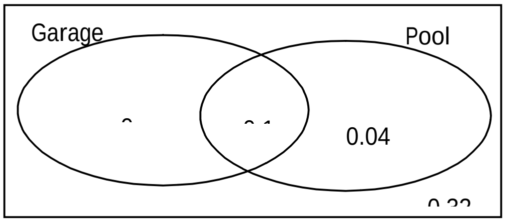


(a) P(Games console and no internet) $= 0.31$

(b) P(Games console or internet but not both)
$= 0.31 + 0.17 = 0.48$

(c) P(neither a games console nor internet access) $= 0.31$

4.4



(a) P(Pool or a Garage) $= 0.47 + 0.17 + 0.68 = 0.68$

(b) P(neither) $= 0.32$

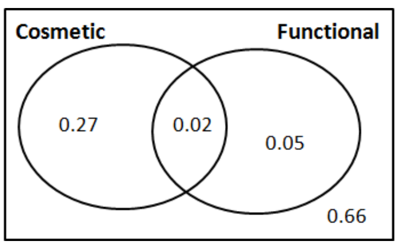
(c) P(Pool but no garage) $= 0.04$

(d) $\text{P(Pool | Garage)} = \dfrac{\text{P(Pool and Garage)}}{\text{P(Garage)}} = \dfrac{0.17}{0.64} \approx 0.266$

(e) Having a pool and a garage are not independent events.
26.6% of homes with garages have pools. Overall,
21% of homes have pools.
If having a garage and a pool were independent these would be the same.

(f) No, having a garage and a pool are not disjoint events.
17% of homes have both.

4.5



(a) $0.27 + 0.2 + 0.5 = 0.34$

(b) 0.27

(c) $P(F \mid C) = \dfrac{P(F \cap C)}{P(C)} = \dfrac{0.02}{0.27 + 0.2} \approx 0.069$

(d) The two kinds of events are not disjoint events,
since 2% of cars have both kinds.

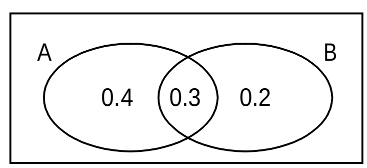
(e) Approximately 6.9% of cars with a cosmetic defects also have functional defects. Overall, the probability that a car has a functional defect is 7%. The probabilities are estimates, so these are probably close enough to say that the two types of defects are independent.

4.6 (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.7 + 0.5 - 0.3$

$P(A \cup B) = 0.9$

or reading from the Venn diagram (without the rule) 0.4+0.3 +0.2=0.9



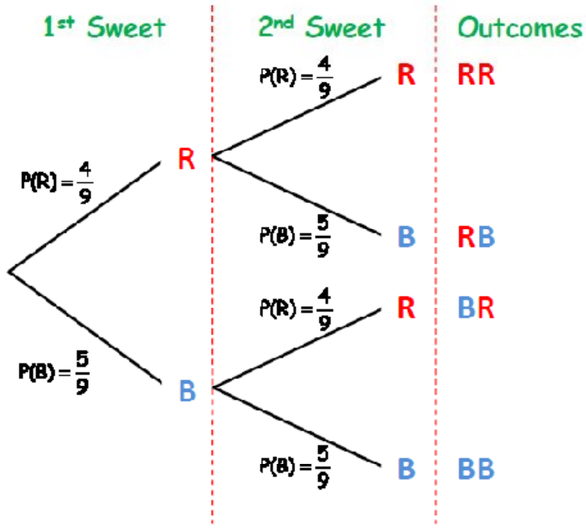
(b) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.3}{0.5} = 0.6$

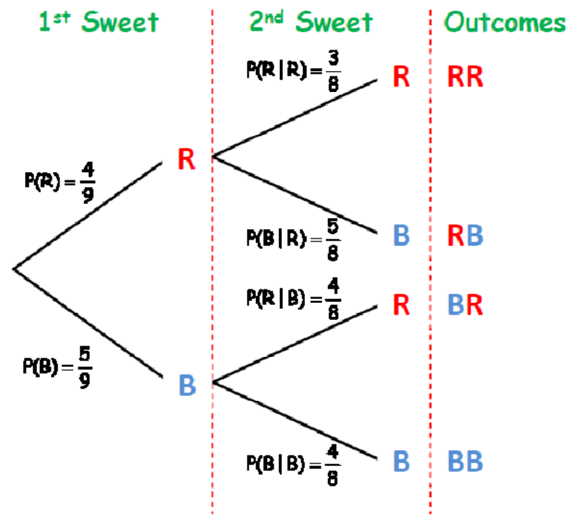
(c) Not independent or Not independent

since $P(A \mid B) \neq P(A)$ since $P(A \cap B) \neq P(A).P(B)$

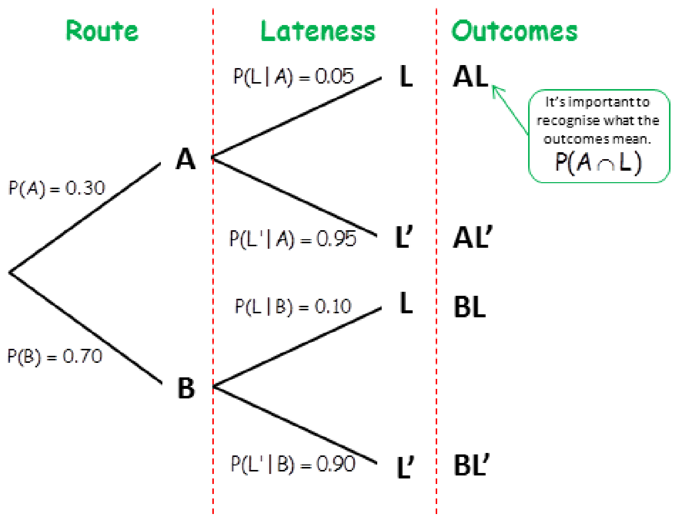
i.e. $0.6 \neq 0.7$ i.e. $0.3 \neq 0.35$

4.7





4.8



(i)

$$P(L) = P(A \cap L) \text{ or } P(B \cap L)$$

$$P(L) = 0.30 \times 0.05 + 0.70 \times 0.10$$

$$P(L) = \frac{17}{200} = 0.085$$

(ii)

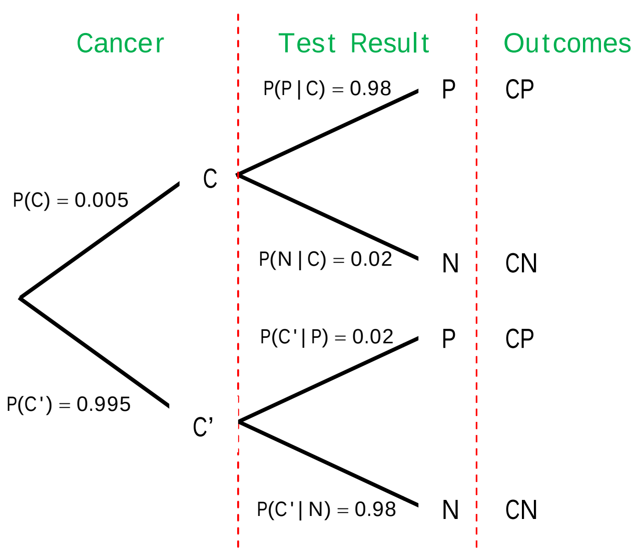
$$P(A \mid L) = \frac{P(A \cap L)}{P(L)}$$

$$P(A \mid L) = \frac{0.30 \times 0.05}{0.085}$$

$$P(A \mid L) = \frac{3}{7} = 0.176 \text{ (3 d.p.)}$$

This shows clearly that $P(A|L) \neq P(L|A)$

4.9



(a)

$$P(P) = P(C \text{ and } P) + P(C' \mid P)$$

$$P(P) = 0.005 \times 0.98 + 0.995 \times 0.02 = 0.0248$$

$$P(P) = 0.0049 + 0.0199 = 0.0248$$

(b)

$$P(C \mid P) = \frac{P(C \cap P)}{P(P)}$$

$$P(C \mid P) = \frac{0.0049}{0.0248} = 0.198$$

When a disease occurs in a very small percentage of the population (in this case, 0.5%), a test that is only 98% accurate will give a lot more false postives than true positives.

In this case 199 false positives for every 49 true positives.

4.10 (a) The net change in your finances is $-€1$ when you lose and €35 when you win.

Outcome	Probability of each outcome, P(x)	Value associated with each outcome(€), x	xP(x)
Get number	$\frac{1}{38}$	+35	$+35\left(\frac{1}{38}\right)=\frac{35}{38}$
Do not get number	$\frac{37}{38}$	–€1	$-1\left(\frac{37}{38}\right)=-\frac{37}{38}$

Expected Value $\mu=\sum xP(x)=\frac{35}{38}-\frac{37}{38}=-\frac{1}{19}\approx-0.0526$

This is not a fair game as the expected value is not zero

(b)

Outcome	Probability of each outcome, P(x)	Value associated with each outcome(€), x	xP(x)
Black	$\frac{18}{38}$	+35	$+35\left(\frac{18}{38}\right)=\frac{315}{19}$
Other colour	$\frac{20}{38}$	–1	$-1\left(\frac{20}{38}\right)=-\frac{20}{38}$

Expected Value $\mu=\sum xP(x)=\frac{315}{19}-\frac{20}{38}=\frac{305}{19}\approx 16.052$

This is not a fair game as the expected value is not zero.